Fein, Burton; **Schacher, Murray**

Properties of iterates and composites of polynomials. (English) Zbl 0865.12003

J. Lond. Math. Soc., II. Ser. 54, No. 3, 489-497 (1996).

Suppose $\mathcal{P}$ is a property of polynomials and r is an arbitrary natural number. This paper is concerned with the following question: does there exist a field K and a polynomial $f(x) \in K[x]$ such that the first r iterates of $f(x)$ have property $\mathcal{P}$ but the next iterate does not? (The iterates of $f(x)$ are defined by $f_1(x) = f(x)$ and $f_{k+1}(x) = f(f_k(x))$ for $k \geq 1$.) The existence of such examples is proven for several of the most frequently considered properties of polynomials: (a) irreducibility, (b) separability, (c) splitting completely, and (d) solvability by radicals. In these examples, K may be taken to be Hilbertian. The question of whether such examples exist over a prescribed Hilbertian field (e.g. $\mathbb{Q}$) is left unresolved.

Reviewer: B.Fein (Corvallis)

MSC:

12E05 Polynomials in general fields (irreducibility, etc.)

Keywords:

composition; irreducibility; polynomials; iterates; Hilbertian field

Full Text: DOI