

Glowinski, Roland; Rieder, Andreas; Wells, Raymond O. jun.; Zhou, Xiaodong
A wavelet multigrid preconditioner for Dirichlet boundary value problems in general domains. (English) [Zbl 0860.65121](#)
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A wavelet-based multigrid method for an elliptic model problem over a square with periodic boundary conditions is introduced. Further, the authors show how this multigrid iteration can be used as a preconditioner for a conjugate gradient method applied to a linear system originating from a wavelet-Galerkin discretization of a Dirichlet boundary value problem via a penalty/fictitious domain formulation. Numerical experiments described in the paper confirm the efficiency of this new iterative solver.

Reviewer: P.Chocholatý (Bratislava)

MSC:

- [65N55](#) Multigrid methods; domain decomposition for boundary value problems involving PDEs Cited in **2** Documents
- [35J25](#) Boundary value problems for second-order elliptic equations
- [65N30](#) Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
- [65F35](#) Numerical computation of matrix norms, conditioning, scaling
- [65F10](#) Iterative numerical methods for linear systems

Keywords:

[numerical examples](#); [wavelet-based multigrid method](#); [preconditioner](#); [conjugate gradient method](#); [wavelet-Galerkin discretization](#); [penalty/fictitious domain formulation](#)

Software:

[Wesseling](#)

Full Text: [DOI](#) [EuDML](#)

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