Alfaro, M.; **López, G.**; **Rezola, M. L.**
Some properties of zeros of Sobolev-type orthogonal polynomials. (English) Zbl 0862.33005
J. Comput. Appl. Math. 69, No. 1, 171-179 (1996).

The authors consider the monic polynomials $Q_n (n = 1, 2, \dots)$ which are orthogonal with respect to a certain inner product involving also a discrete part (Sobolev-type product) of the form (1) $\langle f, g\rangle = \int_I fgd\mu + \sum_{i=0}^{r} M_i f^{(i)}(c)g^{(i)}(c)$, where μ stands for finite positive Borel measure supported on an interval $I \subset \mathbb{R}$ while $c \notin \overset{\circ}{I}$ (the interior of I), $r \geq 1$, $M_i \geq 0$ $(i = 0, \dots, r-1)$ and $M_r > 0$, and derive some properties of their zeros. The results are given in the form of two main theorems. One of them states that Q_n has at least $(n - \overline{n})$ changes of sign in the interior of the convex hull of I, where $\overline{n}$ denotes the number of terms in the discrete part in (1) whose order of derivatives is less than $\overline{n}$. The other main result concerns the interlacing properties of the zeros and gives the conditions under which Q_n and Q_{n+1} have common zeros.

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MSC:

33C45 Orthogonal polynomials and functions of hypergeometric type (Jacobi, Laguerre, Hermite, Askey scheme, etc.)

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