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Nonconvergent bounded trajectories in semilinear heat equations. (English) Zbl 0845.35054

J. Differ. Equations 124, No. 2, 472-494 (1996).

For a bounded domain Ω in $\mathbb{R}^n$ with smooth boundary the following parabolic equation is considered:

$$u_t - \Delta u = g(x,u), \quad x \in \Omega, \quad u(x,t) = 0, \quad x \in \partial\Omega, \tag{1}$$

where $g : (x,t) \in \overline{\Omega} \times \mathbb{R} \to g(x,t) \in \mathbb{R}$ is sufficiently smooth. Solutions of (1) generate a dynamical system on a fractional power space X^α associated with the sectorial operator generated on $X = L^p(\Omega)$, $p > N$, by the differential operator $-\Delta$ with Dirichlet boundary condition on $\partial\Omega$. The dynamical system is gradient like with respect to the usual potential and ω-limit set $\omega(u)$ of an arbitrary bounded solution u of (1) consists only of stationary solutions of (1). If $\omega(u)$ is a one-point set then u is called convergent, otherwise nonconvergent.

Typically, one does not expect any nonconvergent bounded solutions of (1). The goal of this paper was to show that equations (1) with nonconvergent bounded solutions do exist. Actually, it is proved that, with Ω being the unit disk in $\mathbb{R}^2$, there are functions g of class $\mathbb{C}^m$ (with m arbitrary but finite) such that the corresponding parabolic equations (1) possesses a bounded solution whose ω-limit set is diffeomorphic to the unit circle.

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MSC:

35K60 Nonlinear initial, boundary and initial-boundary value problems for linear parabolic equations

35B40 Asymptotic behavior of solutions to PDEs

Keywords:

semilinear heat equation; nonconvergent bounded solutions; ω-limit set

Full Text: DOI