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Finite difference method for generalized Zakharov equations. (English) Zbl 0827.65138

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The authors consider generalized Zakharov equations describing Langmuir waves in plasmas. They develop a conservative difference scheme for the numerical solution of the equations, and point out the invariants associated with the scheme. They show that the truncation error for the scheme is $O(h^2 + \tau^2)$ where h and τ are respectively distance and time steps, thereby improving on previous methods. A discussion is also given of the convergence of the process.

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MSC:

65Z05	Applications to the sciences
65M06	Finite difference methods for initial value and initial-boundary value problems involving PDEs
65M12	Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs
35Q72	Other PDE from mechanics (MSC2000)
82D10	Statistical mechanical studies of plasmas

Keywords:

generalized Zakharov equations; Langmuir waves in plasmas; conservative difference scheme; convergence

Full Text: DOI

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