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**Optimal edge ranking of trees in polynomial time.** (English) Zbl 0826.68093  
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Summary: An edge ranking of a graph is a labeling of the edges using positive integers such that all paths between two edges with the same label contain an intermediate edge with a higher label. An edge ranking is optimal if the highest label used is as small as possible. The edge-ranking problem has applications in scheduling the manufacture of complex multipart products; it is equivalent to finding the minimum height edge-separator tree. In this paper we give the first polynomial-time algorithm to find an optimal edge ranking of a tree, placing the problem in  $\mathcal{P}$ . An interesting feature of the algorithm is an usual greedy procedure that allows us to narrow an exponential search space down to a polynomial search space containing an optimal solution. An  $\mathcal{NC}$  algorithm is presented that finds an optimal edge ranking for trees for constant degree. We also prove that a natural decision problem emerging from our sequential algorithm is  $\mathcal{P}$ -complete.

**MSC:**

**68R10** Graph theory (including graph drawing) in computer science  
**68M20** Performance evaluation, queueing, and scheduling in the context of computer systems

Cited in **19** Documents

**Keywords:**

edge ranking; height edge-separator tree; trees

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