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The cohomology of holomorphic self maps of the Riemann sphere. (English) Zbl 0814.55003
Math. Z. 218, No. 2, 179-190 (1995).

Let Hol_k denote the space of degree k holomorphic self maps of the Riemann sphere, \mathbb{P}^1 , and let $\text{Rat}_k \subset \text{Hol}_k$ denote the subspace of based maps. The cohomology groups $H^*(\text{Rat}_k; \mathbb{Z}_p)$ (p prime) have been computed by *F. R. Cohen, R. L. Cohen, B. M. Mann* and *R. J. Milgram* [*Acta Math.* 166, No. 3/4, 163-221 (1991; [Zbl 0741.55005](#))] and the algebra structure has been given by *B. Totaro* [The cohomology ring of the space of rational functions (preprint MSRI 1990)] for p odd. In this note we compute the cohomology algebra $H^*(\text{Hol}_k; \mathbb{Z}_p)$ when p does not divide k . We also determine the cohomology groups and a graded version of the cohomology algebra when $k = pm$. Direct analysis of the Leray-Serre spectral sequence for the standard bundle $\text{Rat}_k \rightarrow \text{Hol}_k \rightarrow \mathbb{P}^1$ leads to difficulties, and so we make use of the principal bundle $\text{SO}(3) \rightarrow \text{Hol}_k \rightarrow \text{Rat}_k/S^1$. Our computations rely heavily on Milgram's calculation of the groups $H^*(\text{Rat}_k/S^1; \mathbb{Z}_p)$.

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MSC:

- [55N99](#) Homology and cohomology theories in algebraic topology
- [55R20](#) Spectral sequences and homology of fiber spaces in algebraic topology
- [58D15](#) Manifolds of mappings

Cited in **3** Documents

Keywords:

space of degree k holomorphic self maps of the Riemann sphere; cohomology groups; cohomology algebra; Leray-Serre spectral sequence

Full Text: [DOI](#) [EuDML](#)

References:

- CCMM Cohen, F.R., Cohen, R.L., Mann, B.M., Milgram, R.J.: The topology of rational functions and divisors of surfaces. *Acta Math.* 166, 163–221 (1991) · [Zbl 0741.55005](#) · [doi:10.1007/BF02398886](#)
- CLM Cohen, F.R., Lada, T.J., May, J.P.: The homology of iterated loop spaces. (Lect. Notes Math., vol. 533) New York Berlin Heidelberg: Springer 1976 · [Zbl 0334.55009](#)
- CS Cohen, R.L., Shimamoto, D.H.: Rational functions, labelled configurations, and Hilbert schemes. *J. London Math. Soc.* 43, 509–528 (1991) · [Zbl 0756.55005](#) · [doi:10.1112/jlms/s2-43.3.509](#)
- DL Dyer, E., Lashof, R.K.: Homology of iterated loop spaces. *Am. J. Math.* 84, 35–88 (1962) · [Zbl 0119.18206](#) · [doi:10.2307/2372804](#)
- G Guest, M.A.: Topology of the space of absolute minima of the energy functional. *Am. J. of Math.* 106, 21–42 (1984) · [Zbl 0564.58014](#) · [doi:10.2307/2374428](#)
- K Kirwan, F.C.: On spaces of maps from Riemann surfaces to Grassmannians and applications to the cohomology of moduli of vector bundles. *Ark. Mat.* 24, 221–275 (1986) · [Zbl 0625.14026](#) · [doi:10.1007/BF02384399](#)
- MaM1 Mann, B.M., Milgram, R.J.: Some spaces of holomorphic maps to complex Grassmann manifolds. *J. Differ. Geom.* 33, 301–324 (1991) · [Zbl 0736.54008](#)
- MaM2 Mann, B.M., Milgram, R.J.: On the moduli of $\text{SU}(n)$ monopoles and holomorphic maps to flag manifolds. Preprint, University of New Mexico and Stanford University 1991
- May May, J.P.: The geometry of iterated loop spaces. (Lect. Notes Math., vol. 271) New York Berlin Heidelberg: Springer 1972 · [Zbl 0244.55009](#)
- M1 Milgram, R.J.: Iterated loop spaces. *Ann. of Math.* 84, 386–403 (1966) · [Zbl 0145.19901](#) · [doi:10.2307/1970453](#)
- M2 Milgram, R.J.: The structure of spaces of Toeplitz matrices. Preprint, Stanford University and the University of New Mexico 1992
- MiS Milnor, J.W., Stasheff, J.D.: Characteristic classes. (Ann. of Math. Studies, no. 76) Princeton University Press 1974 · [Zbl 0298.57008](#)
- S segal, G.: The topology of spaces of rational functions. *Acta Math.* 143, 39–72 (1979) · [Zbl 0427.55006](#) · [doi:10.1007/BF02392088](#)
- T Totaro, B.: The cohomology ring of the space of rational functions. Preprint, MSRI 1990

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