Talagrand, Michel

On Russo's approximate zero-one law. (English) Zbl 0819.28002

Ann. Probab. 22, No. 3, 1576-1587 (1994).

A subset A of $\{0,1\}^n$ is called monotone if $x_i \leq y_i$, $i = 1, \dots, n$, for $(x_1, \dots, x_n) \in A$ and some $(y_1, \dots, y_n) \in \{0,1\}^n$ implies $(y_1, \dots, y_n) \in A$. Furthermore, the set A_i consists of all $(x_1, \dots, x_n) \in A$ such that $(x_1, \dots, x_{i-1}, 1 - x_i, x_{i+1}, \dots, x_n) \notin A$ is satisfied, $1 \leq i \leq n$, and μ_p stands for the product measure defined by $\mu_p(\{(x_1, \dots, x_n)\}) = p^k(1-p)^{n-k}$, $(x_1, \dots, x_n) \in \{0,1\}^n$, $k = \text{card}\{i \in \{1, \dots, n\}: x_i = 1\}$, $p \in (0,1)$. The following inequality, which is sharp for any $p \in (0,1)$, is the main result:

$$\mu_p(A)(1-\mu_p(A)) \leq K(1-p)\log\frac{2}{p(1-p)}\sum_{i=1}^{n}\frac{\mu_p(A_i)}{\log([(1-p)\mu_p(A_i)]^{-1})}$$

is vaid for all monotone subsets A of $\{0,1\}^n$ and any $p \in (0,1)$, where K is some universal constant.

Reviewer: D.Plachky (Münster)

MSC:

28A35 Measures and integrals in product spaces

60K35 Interacting random processes; statistical mechanics type models; percolation theory

Keywords:

approximate zero-one law; threshold effect; product measure

Full Text: DOI