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The general case of S. Lang's conjecture. (English) Zbl 0823.14009

Cristante, Valentino (ed.) et al., Barsotti symposium in algebraic geometry. Memorial meeting in honor of Iacopo Barsotti, in Abano Terme, Italy, June 24-27, 1991. San Diego, CA: Academic Press. Perspect. Math. 15, 175-182 (1994).

The main result of the present paper is the following theorem on arithmetic abelian varieties:

Let A be an abelian variety over a number field K. Then, for any closed subvariety X of A, there exist finitely many K-rational points $x_i \in X(K)$ and abelian subvarieties $B_i \subset A$, $1 \leq i \leq m$, such that the set $X(K)$ of K-rational points on X is contained in the union $\bigcup_{i=1}^{m}(x_i + B_i(K))$ of translates of K-rational abelian subvarieties of A. In particular, the irreducible components of the Zariski closure $\overline{X(K)}$ of $X(K)$ in X are translates of abelian subvarieties of A.

This theorem had been conjectured by *S. Lang* in 1960 [cf. Publ. Math., Inst. Hautes Étud. Sci. 6, 319–335 (1960; Zbl 0112.13402)]. If X does not contain any positive-dimensional abelian variety, the theorem implies that $X(K)$ is a finite set. This weaker statement, which had also been conjectured by A. Weil, and which may be regarded as a generalization of Mordell's conjecture, was proved by the author in his spectacular paper "Diophantine approximation on abelian varieties" from 1991 [cf. Ann. Math. (2) 133, No. 3, 549–576 (1991; Zbl 0734.14007)].

The general proof of S. Lang's conjecture presented here follows essentially the path used in that previous paper, however after having added a few new ingredients. Among them is, in the first place, a recent result of *D. Abramovich* [cf. "The structure of subvarieties of an abelian variety in arbitrary characteristic" (Preprint, M.I.T. 1990)], which says that, for any closed subvariety X of an abelian variety, the union of all translates of abelian subvarieties contained in X is again closed.

The second main new ingredient concerns some numerical estimates for line bundles on projective schemes over an arbitrary field, which complement the results of R. Hartshorne on ample subvarieties of algebraic varieties [cf. *R. Hartshorne*, "Ample subvarieties of algebraic varieties," Lect. Notes Math. 156 (1970; Zbl 0208.48901)] related to Kleiman's theorem of ampleness. The author points out that these numerical results have been supplied by *J. F. Burnol* in a private letter to him. Finally, the author has added an alternative method of proving the main theorem, which avoids the numerical arguments mentioned above. However, this method is a but more sophisticated and perhaps less transparent.

The paper concludes with some complements concerning applications of the method of proof to the case of function fields. Other applications of the (now proved) conjecture of S. Lang have been discussed in an earlier paper by *M. Hindry* [Invent. Math. 94, No. 3, 575–603 (1988; Zbl 0638.14026)].

For the entire collection see [Zbl 0802.00020].

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MSC:

14G05	Rational points
14K05	Algebraic theory of abelian varieties
14G40	Arithmetic varieties and schemes; Arakelov theory; heights

Keywords:

Lang conjecture; rational point; Mordell conjecture; arithmetic abelian varieties; subvarieties; translates of abelian subvarieties