

Lie, Sophus; Study, Eduard; Engel, Friedrich

Contributions to the theory of differential invariants. Edited and with a foreword and an appendix by G. Czichowski and B. Fritzsche. (Beiträge zur Theorie der Differentialinvarianten. Hrsg. und mit einem Anhang versehen von Günter Czichowski und Bernd Fritzsche.) (German) [Zbl 1107.01313](#)

Teubner-Archiv zur Mathematik 17. Leipzig: Teubner (ISBN 3-8154-2035-0). 228 S. (1993).

This volume contains some important works relating to the development of Lie's theory of differential invariants. This theory (in modern notation) concerns the invariants under the action of a transformation group G (on \mathbb{R}^n) on the jets spaces of $C^\infty(\mathbb{R}^n)$. There are three papers by Lie ("Über Gruppen von Transformationen" [Göttinger Nachr. 1874, 529–542 (1874; [JFM 06.0093.01](#)); also in *Gesammelte Abhandlungen*, Bd. 5, 1–8, Teubner, Leipzig, 1924]; "Über Differentialinvarianten" [Math. Ann. 24, 537–578 (1884; [JFM 16.0091.01](#)); also in *Gesammelte Abhandlungen*, Bd. 6, 95–138, Teubner, Leipzig, 1927]; "Über die Gruppe der Bewegungen und ihre Differentialinvarianten" [Leipz. Ber. 45, 370–378 (1893; [JFM 25.0168.04](#)); also in *Gesammelte Abhandlungen*, Bd. 6, 376–383, Teubner, Leipzig, 1927]).

In reading these one is struck by their motivation in geometric questions. The first paper contains the basic concepts and Lie's first results on transformation groups, while the second is a detailed presentation of the theory/method of differential invariants. The third paper applies the method to the group of motions of \mathbb{R}^3 and, in particular, to the equivalence problem for space curves. There are gaps in this presentation, indicated in Czichowski's mathematical commentary. They are discussed in the fourth paper reprinted here, Study's "Kritische Betrachtungen über Lies Invariantentheorie der endlichen kontinuierlichen Gruppen" [Jber. Deutsch. Math.-Verein. 17, 125–142 (1908; [JFM 39.0206.02](#))], published after Lie's death (1899) from pernicious anemia. Unfortunately Study's own (positive) treatment of the equivalence problem was not reprinted [Trans. Am. Math. Soc. 10, 1–49 (1909; [JFM 40.0658.04](#))]; the comparison would have been very interesting. Lie's long-time collaborator Engel felt that Study had gone too far in his criticism of Lie ("Zu der Studyschen Abhandlung" [Jber. Deutsch. Math.-Verein. 17, 143–144 (1908; [JFM 39.0206.03](#))]), but it was not until 30 years later, after completing the edition of Lie's *Collected works*, that Engel offered his own version of Lie's treatment of the equivalence problem ("Über Lies Invariantentheorie der endlichen kontinuierlichen Gruppen" [Ber. Verh. Sächs. Akad. Wiss. Leipzig, Math.-Phys. Kl. 90, 137–174 (1938; [Zbl 0020.29502](#) and [JFM 64.1096.02](#))]). These two articles by Engel form the fifth and seventh papers reprinted in this volume. Note that the latter of these two articles was published eight years after Study's death.

The volume also includes a very interesting general exposition of Lie's theory by Engel, originally published as the Editor's Foreword to Band 6 of Lie's *Gesammelte Abhandlungen*. The book also contains portraits of the three authors, some facsimile documents, the mathematical commentaries by Czichowski referred to above and an extensive biographical commentary by Fritzsche. The last of these relies on letters and obituaries (contemporary), and is quite interesting in its own right.

Reviewer: [Jeffrey S. Joel \(Bloomington\)](#)

MSC:

- [01A75](#) Collected or selected works; reprintings or translations of classics
- [01A70](#) Biographies, obituaries, personalia, bibliographies
- [22-03](#) History of topological groups
- [37-03](#) History of dynamical systems and ergodic theory
- [53-03](#) History of differential geometry
- [58-03](#) History of global analysis