

**Boyer, C. P.; Mann, B. M.; Hurtubise, J. C.; Milgram, R. J.**

**The topology of the space of rational maps into generalized flag manifolds.** (English)

Zbl 0844.57037

Acta Math. 173, No. 1, 61-101 (1994).

This paper deals with a complex semi-simple Lie group  $G$  and parabolic subgroup  $P$ . The space  $\text{Rat}(G/P)$  of holomorphic maps from  $\mathbb{P}^1$  to  $G/P$  has components  $\text{Rat}_{\mathbf{k}}(G/P)$  labelled by a multidegree  $\mathbf{k} = (k_1, \dots, k_n) \in \pi_2(G/P)$ , whose  $j$ th component  $k_j$  is the intersection number of the image of  $S^2$  with the closure of a codimension-1 Bruhat cell. The components of  $\Omega^2(G/P)$  can be labelled similarly, and there is an inclusion  $\text{Rat}_{\mathbf{k}}(G/P) \rightarrow \Omega_{\mathbf{k}}^2(G/P)$ . The main theorem states that this map is a homotopy equivalence through dimension  $cl(\mathbf{k}) - 1$  or  $cl(\mathbf{k}) - 2$ , where  $l(\mathbf{k}) = \min(k_i)$  and  $c \leq \frac{1}{2}$  is a positive constant which depends only on  $G/P$ . The proof involves analyzing homology groups and fundamental groups. The constant  $c$  depends on codimensions in a stratification of  $\text{Rat}_{\mathbf{k}}(G/P)$ .

Reviewer: D.Davis (Bethlehem)

**MSC:**

57R99 Differential topology

Cited in **1** Review  
Cited in **13** Documents

**Keywords:**

rational maps; loop spaces; generalized flag manifolds

**Full Text:** DOI

**References:**

- AHS Atiyah, M. F., Hitchin, N. J. & Singer, I., Self-duality in four dimensional Riemannian geometry. Proc. Roy. Soc. London Ser. A, 362 (1978), 425–461. · Zbl 0389.53011 · doi:10.1098/rspa.1978.0143
- AJ Atiyah, M. F. & Jones, J. D., Topological aspects of Yang-Mills theory. Comm. Math. Phys., 61 (1978), 97–118. · Zbl 0387.55009 · doi:10.1007/BF01609489
- BE Baston, R. J. & Eastwood, M. G., The Penrose Transform. Oxford University Press, 1989. · Zbl 0726.58004
- BHMM Boyer, C. P., Hurtubise, J. C., Mann, B. M. & Milgram, R. J., The topology of instanton moduli spaces. I: The Atiyah-Jones conjecture. Ann. of Math., 137 (1993), 561–609. · Zbl 0816.55002 · doi:10.2307/2946532
- BM Boyer, C. P. & Mann, B. M., Monopoles, non-linear  $\{\sigma\}$  models, and two-fold loop spaces. Comm. Math. Phys., 115 (1988), 571–594. · Zbl 0656.58049 · doi:10.1007/BF01224128
- Bo Borel, A., Kählerian coset spaces of semisimple Lie groups. Proc. Nat. Acad. Sci. U.S.A., 40 (1954), 1147–1151. · Zbl 0058.16002 · doi:10.1073/pnas.40.12.1147
- C2M2 Cohen, F. R., Cohen, R. L., Mann, B. M. & Milgram, R. J., The topology of rational functions and divisors of surfaces. Acta Math., 166, (1991), 163–221. · Zbl 0741.55005 · doi:10.1007/BF02398886
- D Donaldson, S. K., Nahm's equations and the classification of monopoles. Comm. Math. Phys., 96 (1988), 387–407. · Zbl 0603.58042 · doi:10.1007/BF01214583
- Gr Gravesen, J., On the topology of spaces of holomorphic maps. Acta Math., 162 (1989), 247–286. · Zbl 0696.58014 · doi:10.1007/BF02392839
- Gul Guest, M. A., Topology of the space of absolute minima of the energy functional. Amer. J. Math., 106 (1984), 21–42. · Zbl 0564.58014 · doi:10.2307/2374428
- Gu2 Guest, M. A. The topology of the space of rational curves on a toric variety. Preprint, Rochester University, 1993.
- HM1 Hurtubise, J. C. & Murray, M. K., On the construction of monopoles for the classical groups. Comm. Math. Phys., 122 (1989), 35–89. · Zbl 0682.32026 · doi:10.1007/BF01221407
- HM2 – Monopoles and their spectral data. Comm. Math. Phys., 133 (1990), 487–508. · Zbl 0708.58035 · doi:10.1007/BF02097006
- Hum Humphreys, J. E., Linear Algebraic Groups. Springer-Verlag, 1975.
- Hur1 Hurtubise, J. C., Instantons and jumping lines. Comm. Math. Phys., 105 (1986), 107–122. · Zbl 0596.32031 · doi:10.1007/BF01212344
- Hur2 – The classification of monopoles for the classical groups. Comm. Math. Phys., 120 (1989), 613–641. · Zbl 0824.58015 · doi:10.1007/BF01260389

- I Itaka, S., Algebraic Geometry Springer-Verlag, New York, 1982.
- Ki1 Kirwan, F. C., On spaces of maps from Riemann surfaces to Grassmannians and applications to the cohomology of moduli of vector bundles. *Ark. Mat.*, 24 (1986), 221–275. · [Zbl 0625.14026](#) · [doi:10.1007/BF02384399](#)
- Ki2 Kirwan, F. C. Geometric invariant theory and the Atiyah-Jones conjecture. Preprint, Oxford University, 1992.
- Ko Kodaira, K., A theorem of completeness of characteristic systems for analytic families of compact submanifolds of complex manifolds. *Ann. of Math.*, 75 (1962), 146–162. · [Zbl 0112.38404](#) · [doi:10.2307/1970424](#)
- Mc McDuff, D., Configuration spaces of positive and negative particles. *Topology*, 14 (1975), 91–107. · [Zbl 0296.57001](#) · [doi:10.1016/0040-9383\(75\)90038-5](#)
- MF Mumford, D. & Fogarty, J., Geometric Invariant Theory, 2nd edition. Springer-Verlag, New York, 1982. · [Zbl 0504.14008](#)
- Mi Milgram, R. J., Iterated loop spaces. *Ann. of Math.*, 84 (1966), 386–403. · [Zbl 0145.19901](#) · [doi:10.2307/1970453](#)
- MM1 Mann, B. M. & Milgram, R. J., Some spaces of holomorphic maps to complex Grassmann manifolds. *J. Differential Geom.*, 33 (1991), 301–324. · [Zbl 0736.54008](#)
- MM2 – On the moduli space of  $SU(n)$  monopoles and holomorphic maps to flag manifolds. *J. Differential Geom.*, 38 (1993), 39–103. · [Zbl 0790.53053](#)
- MM3 – The topology of rational maps to Grassmannians and a homotopy theoretic proof of the Kirwan stability theorem. *Contemp. Math. AMS*, 146 (1993), 251–275. · [Zbl 0795.55008](#)
- Mu Murray, M. K., Stratifying monopoles and rational maps. *Comm. Math. Phys.*, 125 (1989), 661–674. · [Zbl 0693.32012](#) · [doi:10.1007/BF01228347](#)
- RR Ramanan, S. & Ramanathan, A., Projective normality of flag varieties and Schubert varieties. *Invent. Math.*, 79 (1985), 217–224. · [Zbl 0553.14023](#) · [doi:10.1007/BF01388970](#)
- S Segal, G., The topology of rational functions. *Acta Math.*, 143 (1979), 39–72. · [Zbl 0427.55006](#) · [doi:10.1007/BF02392088](#)
- Ta1 Taubes, C. H., Monopoles and maps from  $S^2$  to  $S^2$ ; the topology of configuration space. *Comm. Math. Phys.*, 95 (1984), 345–391. · [Zbl 0594.58053](#) · [doi:10.1007/BF01212403](#)
- Ta2 – The stable topology of self-dual moduli spaces. *J. Differential Geom.*, 29 (1989), 163–230. · [Zbl 0669.58005](#)
- Ti Tian, Y., The based  $SU(n)$ -instanton moduli spaces. *Math. Ann.*, 298 (1994), 117–139. · [Zbl 0788.32013](#) · [doi:10.1007/BF01459729](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.