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The nonexistence of reduction rules giving an embedding into a k-tree. (English)

Zbl 0809.05034
Discrete Appl. Math. 54, No. 2-3, 219-223 (1994).

This paper considers the k-tree embedding problem: given a graph $G = (V, E)$, is G a subgraph of a k-tree and if so, find such an embedding. This is equivalent to determining whether G has treewidth at most k, and if so, finding a tree-decomposition with optimal width of G.

A locally triggered reduction rule, applied to a graph G, takes a vertex v which belongs to a subgraph of G with a certain well- described structure, connects all neighbors of v, and removes v. For $k = 2$, $k = 3$, a set of such reduction rules exist, such that the rule can be applied repeatedly until the empty graph results, if and only if G is in the class to be recognized. These sets can be used to obtain linear time algorithms for the k-embedding problem.

This paper shows that for $k = 4$, such reduction rules do not exist. More general sets of reduction rules for the k-tree embedding problem (for arbitrary fixed k) are known to exist, see *S. Arnborg, D. G. Corneil* and *A. Proskurowski* [SIAM J. Algebraic Discrete Methods 8, 277-284 (1987; Zbl 0611.05022)]. Also, linear time algorithms for the k-embedding problem, using a different method, are known (k fixed), see *H. L. Bodlaender* [A linear time algorithm for finding tree- decompositions of small treewidth, Proc. 25th Ann. Symp. Theor. Comp. Sci., 226-234 (1993)].

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MSC:

05C05	Trees
05C10	Planar graphs; geometric and topological aspects of graph theory
68R10	Graph theory (including graph drawing) in computer science

Keywords:

k-tree embedding problem; embedding; treewidth; tree-decomposition; triggered reduction rule; linear time algorithms

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