Dostoglou, Stamatis; Salamon, Dietmar A.
**Self-dual instantons and holomorphic curves.** (English) Zbl 0812.58031
Ann. Math. (2) 139, No. 3, 581-640 (1994).

Let $P \to \Sigma$ be a nontrivial SO(3)-bundle over a compact oriented Riemann surface of genus at least 2 and let $f : P \to P$ be an automorphism, $h : \Sigma \to \Sigma$ the induced diffeomorphism. In this situation two Floer type homology groups can be constructed. First one considers the moduli space $\mathcal{M}(P)$ of flat connections on $P$. This is a simply connected compact manifold with $\pi_2(\mathcal{M}(P)) \cong \mathbb{Z}$. It carries a natural symplectic structure and $f$ induces a symplectomorphism $\phi_f : \mathcal{M}(P) \to \mathcal{M}(P)$. Extending a construction of *A. Floer* [Commun. Math. Phys. 120, No. 4, 575-611 (1989; Zbl 0755.58022)] one can define a chain complex generated by the fixed points of $\phi_f$, provided these are nondegenerate; otherwise one passes to a perturbation of $\phi_f$. These fixed points are the critical points of the (perturbed) symplectic action functional. The boundary operator is determined by certain gradient flow lines of this functional (holomorphic curves). The homology groups of this chain complex are denoted by $HF_*^{\text{symp}}(\mathcal{M}(P), \phi_f)$.

One can also consider the induced bundle $P_f \to \Sigma_h$ where $P_f$ and $\Sigma_h$ are the mapping cylinders of $f$ and $h$, respectively. This is a principal SO(3)-bundle over a compact oriented 3-manifold. The (perturbed) Chern-Simons functional can be used to define a chain complex generated by the critical points (flat connections modulo gauge equivalence). The boundary operator is again determined by certain gradient flow lines (self-dual Yang-Mills instantons on the 4-manifold $\Sigma_h \times \mathbb{R}$); cf. *A. Floer* [Commun. Math. Phys. 118, No. 2, 215-240 (1988; Zbl 0684.53027)]. The homology groups of this chain complex are denoted by $HF_*^{\text{inst}}(\Sigma_h, P_f)$.

Main Theorem. There is a natural isomorphism of Floer homologies

$$HF_*^{\text{inst}}(\Sigma_h, P_f) \cong HF_*^{\text{symp}}(\mathcal{M}(P), \phi_f).$$

In particular, for $f = \text{id} : H_*^{\text{inst}}(\Sigma \times S^1, P \times S^1) \cong HF_*^{\text{symp}}(\mathcal{M}(P), \mathbb{Z})$.

The critical points of the two functionals can be identified. In an earlier paper the authors showed that the gradings of the chain complexes coincide. Thus the proof of the main theorem consists of comparing the boundary operators. The idea is to approximate holomorphic curves by self-dual instantons.

Reviewer: Th.J.Bartsch (Heidelberg)

**MSC:**

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|---|---|
| 37J99 | Dynamical aspects of finite-dimensional Hamiltonian and Lagrangian systems |
| 57N10 | Topology of general 3-manifolds (MSC2010) |
| 58E05 | Abstract critical point theory (Morse theory, Lyusternik-Shnirel'man theory, etc.) in infinite-dimensional spaces |



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Floer homology; instanton homology; holomorphic curves; instantons

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