

**Voisin, Claire**

**On the Abel-Jacobi map for Calabi-Yau threefolds. (Sur l'application d'Abel-Jacobi des variétés de Calabi-Yau de dimension trois.)** (French) Zbl 0808.14030

*Ann. Sci. Éc. Norm. Supér.* (4) 27, No. 2, 209-226 (1994).

The aim of this paper is to prove the following theorem: Let  $X$  be a Calabi-Yau threefold and let  $\{X_t\}$  be a generic deformation of  $X$ , then the Abel-Jacobi map  $\Phi_{X_t}$  is nonzero modulo torsion.

The methods and the results of *C. Voisin* [*Int. J. Math.* 3, No. 5, 699-715 (1992; [Zbl 0772.14015](#))] are used and extended here. Let  $\Sigma$  be a smooth surface of a suitable degree in  $X$  and let  $(X_t, \Sigma_t)$  be a deformation of the pair  $(X, \Sigma)$ . The proof is based on a description of the vanishing of  $\Phi_{X_t}$  for a general  $t$  in terms of the variation of the mixed Hodge structure of  $X - \Sigma$ , and leads to the study of a system of quadrics whose dimension is bounded. Then the result follows by contradiction.

Reviewer: [L. Picco Botta \(Torino\)](#)

**MSC:**

- [14J30](#) 3-folds
- [14D07](#) Variation of Hodge structures (algebraic-geometric aspects)
- [14J32](#) Calabi-Yau manifolds (algebraic-geometric aspects)

Cited in **1** Review  
Cited in **2** Documents

**Keywords:**

Calabi-Yau threefold; generic deformation; Abel-Jacobi map; variation of mixed Hodge structure

**Full Text:** [DOI](#) [Numdam](#) [EuDML](#)

**References:**

- [1] A. ALBANO , Infinite Generation of the Griffiths Group : a Local Proof (Thesis, University of Utah, 1986 ).
- [2] F. BARDELLI , Curves of Genus Three on a General Abelian Threefold and the Non Finite Generation of the Griffiths Group , dans : Arithmetic of complex manifolds ; Lecture Notes in Math., n<sup>o</sup> 1399 Springer, 1989 , p. 10-26. MR 91a:14004 | Zbl 0703.14004 · [Zbl 0703.14004](#)
- [3] F. BARDELLI et S. MÜLLER-STACH , Algebraic Cycles on Certain Calabi-Yau Threefolds , preprint de l'Université de Pise, 1992 . · [Zbl 0791.14017](#)
- [4] H. CLEMENS , Homological Equivalence, Modulo Algebraic Equivalence, is Not Finitely Generated (*Inst. Hautes Études Sci., Publ. Math.*, vol. 58, 1983 , p. 19-38). Numdam | MR 86d:14043 | Zbl 0529.14002 · [Zbl 0529.14002](#) · numdam:PMIHES\_1983\_\_58\_\_19\_0 · eudml:103992
- [5] H. CLEMENS , A Note on Some Formal Properties of the Infinitesimal Abel-Jacobi Mapping , dans *Geometry today*, E. ARBARELLO et al. éd. (*Progress in Math.*, vol. 60, Birkhäuser, 1985 , p. 69-73). MR 88g:14042 | Zbl 0569.14020 · [Zbl 0569.14020](#)
- [6] P. DELIGNE , Théorie de Hodge II (*Inst. Hautes Études Sci., Publ. Math.*, vol. 40, 1971 , p. 5-58). Numdam | MR 58 #16653a | Zbl 0219.14007 · [Zbl 0219.14007](#) · doi:10.1007/BF02684692 · numdam:PMIHES\_1971\_\_40\_\_5\_0 · eudml:103914
- [7] F. ELZEIN et S. ZÜCKER , Extendability of Normal Functions Associated to Algebraic cycles , dans : *Topics in Transcendental Algebraic Geometry*, P. GRIFFITHS éd. (*Ann. of Math. Studies*, Study, 106, Princeton University Press, 1984 , p. 269-288). MR 756857 | Zbl 0545.14017 · [Zbl 0545.14017](#)
- [8] R. FRIEDMAN , On Threefolds with Trivial Canonical Bundle , preprint . · [Zbl 0753.14035](#)
- [9] M. GREEN , Griffiths' Infinitesimal Invariant and the Abel-Jacobi Map (*J. Differential Geom.*, vol. 29, 1989 , p. 545-555). MR 90c:14006 | Zbl 0692.14003 · [Zbl 0692.14003](#)
- [10] M. GREEN , The Period Map for Hypersurface Sections of High Degree of an Arbitrary Variety (*Compositio Math.*, vol. 55, 1985 , p. 135-156). Numdam | MR 87b:32038 | Zbl 0588.14004 · [Zbl 0588.14004](#) · numdam:CM\_1985\_\_55\_2\_135\_0 · eudml:89712
- [11] P. GRIFFITHS , Infinitesimal Variations of Hodge Structure III : Determinantal Varieties and the Infinitesimal Invariant of Normal Functions (*Compositio Math.*, vol. 50, 1983 , p. 267-324). Numdam | MR 86e:32026c | Zbl 0576.14009 · [Zbl 0576.14009](#) · numdam:CM\_1983\_\_50\_2-3\_267\_0 · eudml:89626
- [12] P. GRIFFITHS , Periods of Integrals on Algebraic Manifolds I, II (*Amer. J. Math.*, vol. 90, 1968 , p. 568-626, 805-865). MR 37 #5215 | Zbl 0183.25501 · [Zbl 0183.25501](#) · doi:10.2307/2373485

- [13] P. GRIFFITHS , On the Periods of Certain Rational Integrals I, II (Ann. of Math., vol. 90, 1969 , p. 460-541). MR 41 #5357 | Zbl 0215.08103 · Zbl 0215.08103 · doi:10.2307/1970746
- [14] N. KATZ , The Regularity Theorem in Algebraic Geometry , dans : Actes du congrès international de Mathématiques, Nice, 1970 , tome 1, p. 437-443. MR 57 #12512 | Zbl 0235.14006 · Zbl 0235.14006
- [15] S. O. KIM , Noether-Lefschetz Locus for Surfaces (Trans. Amer. Math. Soc., vol. 324, n<sup>o</sup> 1, 1991 ). MR 91f:14031 | Zbl 0739.14019 · Zbl 0739.14019 · doi:10.2307/2001513
- [16] M. NORI , Cycles in the Generic Abelian Threefold (Proc. Indian Acad. Sci., vol. 99, 1989 , p. 191-196). MR 90k:14003 | Zbl 0725.14006 · Zbl 0725.14006 · doi:10.1007/BF02864390
- [17] K. PARANJAPE , Curves on Threefolds with Trivial Canonical Bundle (Proc. Indian Acad. Sci. Math. vol. 101, 1991 , n<sup>o</sup> 3, p. 199-213). MR 93a:14036 | Zbl 0759.14031 · Zbl 0759.14031 · doi:10.1007/BF02836802
- [18] C. VOISIN , Une remarque sur l'invariant infinitésimal des fonctions normales (C. R. Acad. Sci. Paris, vol. 307, Série I, 1988 , p. 157-160). MR 90a:14007 | Zbl 0673.14005 · Zbl 0673.14005
- [19] C. VOISIN , Une approche infinitésimale du théorème de H. Clemens sur les cycles d'une quintique générale de  $\mathbb{P}^4$  (J. Algebraic Geometry, vol. I, 1992 , p. 157-174). MR 92k:14008 | Zbl 0787.14003 · Zbl 0787.14003
- [20] C. VOISIN , Densité du lieu de Noether-Lefschetz pour les sections hyperplanes des variétés de Calabi-Yau (International J. of Math. vol. 3, n<sup>o</sup> 5, 1992 , p. 699-715). MR 93m:14034 | Zbl 0772.14015 · Zbl 0772.14015 · doi:10.1142/S0129167X92000345
- [21] J. STEENBRINK et S. ZÜCKER , Variation of Mixed Hodge Structure I (Invent. Math., vol. 80, p. 489-542). MR 87h:32050a | Zbl 0626.14007 · Zbl 0626.14007 · doi:10.1007/BF01388729 · eudml:143242

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.