

Silverman, Joseph H.

Integer points, diophantine approximation, and iteration of rational maps. (English)

Zbl 0811.11052

Duke Math. J. 71, No. 3, 793-829 (1993).

Let $\varphi(z) \in \mathbb{Q}(z)$ be a rational function with rational coefficients. Then φ determines an endomorphism of $\mathbb{P}^1(\mathbb{Q})$; this in turn gives rise to a dynamical system. Interest in such systems stems from φ -canonical heights, as studied by *G. S. Call* and the author [Compos. Math. 89, 163-205 (1993)].

This paper studies the diophantine properties of orbits under such systems. For example: Theorem. If $\varphi(z)$ has degree at least 2 and if $\varphi \circ \varphi \notin \mathbb{Q}[z]$ then for any $t \in \mathbb{P}^1(\mathbb{Q})$ the orbit $\{t, \varphi(t), \varphi(\varphi(t)), \varphi(\varphi(\varphi(t))), \dots\}$ contains only finitely many distinct integers.

This can often be made more quantitative, as follows. Theorem. If, furthermore, $1/(\varphi \circ \varphi) \notin \mathbb{Q}[1/z]$ and the orbit is nonrepeating, then writing the n -th iterate of t as $\varphi^n(t) = a_n/b_n$ in lowest terms, we have $\lim_{n \rightarrow \infty} (|a_n|/|b_n|) = 1$.

Finally, this paper proves that, in many cases, it is possible to show that orbits do not get extremely close to ∞ in many cases (described precisely in the paper), in the sense that

$$\lim_{n \rightarrow \infty} \frac{\delta(A, \varphi^n(t))}{(\deg \varphi)^n} = 0$$

for fixed $A \in \mathbb{P}^1(\mathbb{C})$, where δ denotes a distance function on $\mathbb{P}^1(\mathbb{C})$.

The methods of the paper consist of reducing to diophantine equations such as Thue equations or more generally those covered by Siegel's theorem. Therefore some results are ineffective in the sense that some constants cannot be explicitly computed.

Reviewer: P.Vojta (Berkeley)

MSC:

- 11J99 Diophantine approximation, transcendental number theory
- 14G25 Global ground fields in algebraic geometry
- 30D05 Functional equations in the complex plane, iteration and composition of analytic functions of one complex variable
- 37B99 Topological dynamics

Cited in **6** Reviews
Cited in **34** Documents

Keywords:

iteration of rational maps; integral point; dynamical system; φ - canonical heights; diophantine properties of orbits; orbit; diophantine equations; Thue equations; Siegel's theorem

Full Text: [DOI](#)

References:

- [1] A. Beardon, Iteration of Rational Functions , Graduate Texts in Math., vol. 132, Springer-Verlag, New York, 1991. · Zbl 0742.30002
- [2] G. Call and J. Silverman, Canonical heights on varieties with morphisms , to appear in Compositio Math. · Zbl 0826.14015 · numdam:CM_1993__89_2_163_0 · eudml:90256
- [3] R. Devaney, An Introduction to Chaotic Dynamical Systems , Addison-Wesley Studies in Nonlinearity, Addison-Wesley, Redwood City, Calif., 1989. · Zbl 0695.58002
- [4] S. Lang, Elliptic Curves: Diophantine Analysis , Grundlehren Math. Wiss., vol. 231, Springer-Verlag, Berlin, 1978. · Zbl 0388.10001
- [5] S. Lang, Fundamentals of Diophantine Geometry , Springer-Verlag, New York, 1983. · Zbl 0528.14013
- [6] W. Rudin, Real and Complex Analysis , second ed., McGraw-Hill, New York, 1974. · Zbl 0278.26001
- [7] J. H. Silverman, Integer points on curves of genus ≥ 1 , J. London Math. Soc. (2) 28 (1983), no. 1, 1-7. · Zbl 0487.10015 ·

[doi:10.1112/jlms/s2-28.1.1](https://doi.org/10.1112/jlms/s2-28.1.1)

- [8] J. H. Silverman, *The Arithmetic of Elliptic Curves*, Graduate Texts in Math., vol. 106, Springer-Verlag, New York, 1986. ·
Zbl 0585.14026

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.