Aspinwall, Paul S.; Greene, Brian R.; Morrison, David R.
The monomial-divisor mirror map. (English) Zbl 0798.14030
Int. Math. Res. Not. 1993, No. 12, 319-337 (1993).

Mirror symmetry phenomenon proposes to link the conformal field theories associated to a pair of Calabi-Yau manifolds. *S.-S. Roan* [J. Math. 2, No. 4, 439-455 (1991)] related the phenomenon to certain duality in toric geometry in the case of Calabi-Yau hypersurfaces in weighted projective spaces, where concrete realization of mirror symmetry had been given earlier by *B. R. Greene* and *M. R. Plesser* ["Duality in Calabi-Yau moduli space", Nuclear Phys. B 338, No. 1, 15-37 (1990)].

More generally, *V. V. Batyrev* ["Dual polyhedra and mirror symmetry for Calabi-Yau hypersurfaces in toric varieties", Duke Math. J. (to appear)] considered Calabi-Yau hypersurfaces in toric Fano varieties corresponding to reflexive integral polytopes as follows: An integral convex polytope P with respect to a lattice M in a real affine space $M_{\mathbb{R}} := M \otimes \mathbb{R}$ is a convex hull of a finite number of points in M. P is said to be reflexive if the origin is in the interior of P and if each affine hyperplane $H \subset M_{\mathbb{R}}$ which meets P in a face of codimension one has the form $H = \{m \in M_{\mathbb{R}} \mid \langle n_H, m\rangle = -1\}$ for some n_H in the dual lattice $N := \mathrm{Hom}(M, \mathbb{Z})$. P gives rise to a toric Fano variety V (with at worst Gorenstein canonical singularities), and a general anticanonical divisor X is a Calaby-Yau variety (with at worst Gorenstein singularities). The polar polyhedron $P^\circ := \{n \in N_{\mathbb{R}} \mid \langle m, n\rangle \geq -1$, for all $m \in M_{\mathbb{R}}\}$ which is the convex hull in $N_{\mathbb{R}}$ of the n_H's corresponding to the supporting hyperplanes H for codimension-one faces of P, is a reflexive integral polytope in $N_{\mathbb{R}}$ with respect to the dual lattice N. Batyrev proposed that X together with a general anticanonical divisor Y of the toric Fano variety V° corresponding to P° should form a mirror pair.

The authors show the existence of an isomorphism between certain Hodge groups associated to X and Y, which is an important piece of evidence for the full mirror symmetry relationship between X and Y linking the corresponding conformal field theories. More specifically, let $\widehat{X} \to X$ and $\widehat{Y} \to Y$ be partial resolutions of singularities. Batyrev constructed an isomorphism between the Hodge groups $H^{1,2}(\widehat{X})$ and $H^{d-1,1}(\widehat{Y})$ as predicted by mirror symmetry, where $d = \dim X = \dim Y$.

Under a certain dominance assumption, the authors here construct a natural isomorphism, called the monomial-divisor mirror map, from the space $H^{1,1}_{\text{toric}}(\widehat{X})$ spanned by the divisor classes coming from the ambient toric variety V to the space $H^{d-1,2}_{\text{poly}}(\widehat{Y})$ of first-order polynomial deformations of Y in V°. The authors then formulate a very precise conjecture about the form of the mirror symmetry. One of its consequences is the authors' earlier conclusion in their paper "Multiple mirror manifolds and topology change in string theory", Phys. Lett. B 303, No. 3/4, 249-259 (1993) that the Kähler moduli space of the nonlinear sigma models whose targets are different but mutually birational partial resolutions of singularities $\widehat{X}$ should be connected by analytic continuation.

Among other things, the following recent developments in toric geometry play important roles in the authors' arguments: *D. A. Cox*, "The homogeneous coordinate ring of a toric variety", J. Algebr. Geom. (to appear), *I. M. Gel'fand*, *A. V. Zelevinskij* and *M. M. Kapranov*, Leningr. Math. J. 2, No. 3, 449-505 (1991); translation from Algebra Anal. 2, No. 3, 1-62 (1990; Zbl 0741.14033) and *T. Oda* and *H. S. Park*, Tôhoku Math. J., II. Ser. 43, No. 3, 375-399 (1991; Zbl 0782.52006).

Reviewer: T.Oda (Sendai)

MSC:

14M25	Toric varieties, Newton polyhedra, Okounkov bodies
81T40	Two-dimensional field theories, conformal field theories, etc. in quantum mechanics
14J45	Fano varieties
32J81	Applications of compact analytic spaces to the sciences
53C55	Global differential geometry of Hermitian and Kählerian manifolds
32G05	Deformations of complex structures

Keywords:

mirror symmetry; Calabi-Yau hypersurfaces in toric Fano varieties; reflexive integral polytopes; conformal field theories; monomial-divisor mirror map

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