Jordan, David A.

Height one prime ideals of certain iterated skew polynomial rings. (English) Zbl 0804.16028

Math. Proc. Camb. Philos. Soc. 114, No. 3, 407-425 (1993).

The author studies the height-1 prime ideals of the ring R described below. This very interesting but rather complicated paper contains much more than is covered by the following summary. Let A be a commutative integral domain which is finitely-generated as an algebra over an algebraically-closed field k; let α be a k-automorphism of A; and let u and ρ be fixed non-zero elements of A and k respectively. The ring R is generated over A by x and y subject to the relations: $xy-\rho yx=u-\rho\alpha(u)$; $xa=\alpha^{-1}(a)x$ and $ya=\alpha(a)y$ for all a in A. Alternatively, it can be constructed in two stages as an iterated skew polynomial ring in x and y over A. The author previously studied R when $\rho=1$, but this new wider context includes additional important examples such as the quantized Weyl algebra. Many of the results include the further assumptions that $A\neq k$ and that A is α-simple, and we shall make these assumptions from now on. A complete description is given of all the height-1 prime ideals of R, together with generators for those which are principal. All the principal height-1 primes of R are shown to be primitive (with corresponding simple modules constructed explicitly), and if A has Krull dimension 1 then every height-1 prime of R is principal and primitive.

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MSC:

16S36	Ordinary and skew polynomial rings and semigroup rings
16D25	Ideals in associative algebras
16U20	Ore rings, multiplicative sets, Ore localization
16D60	Simple and semisimple modules, primitive rings and ideals in associative algebras

Keywords:
height-1 prime ideals; integral domain; iterated skew polynomial ring; quantized Weyl algebra; generators; principal height-1 primes; simple modules

Full Text: DOI

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