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**Combinatorial operations on near-triangulations of the plane.** (Chinese. English summary)

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Summary: In this paper combinatorial operations,  $T^*$ ,  $T^+$  and  $\pi$ , on near-triangulations are introduced and used in a process of building up a given near-triangulation  $G$  bounded by a circuit  $Q_r$ . In this process one starts from an arbitrary triangle  $\Delta$ , and adds a new triangle  $\Delta_{i+1}$ , at each time, to the intermediate near-triangulation  $G_i$  previously formed so that one or two properly assigned sides on the bounding circuit of  $G_i$  is or are coincident with that of  $\Delta_{i+1}$ . At the end of this process one gets  $G$ .

Based on the above combinatorial results, conjectures which are concerned only with the properties of 4-colorings of circuits and each of which is equivalent to the Four-Color Theorem are given in the present paper. It is also pointed out that an enlightening conjecture of the above type—a conjecture at the end of the paper [*H. Whitney* and *W. T. Tutte*, *Util. Math.* 2, 241-281 (1972; [Zbl 0253.05120](#))] is not true even for circuits of length 4.

**MSC:**

[05C15](#) Coloring of graphs and hypergraphs

[05C99](#) Graph theory

**Keywords:**

plane; four-color theorem; combinatorial operations; near-triangulations; circuit; triangle; 4-colorings