Genest, Christian; **Rivest, Louis-Paul**

Statistical inference procedures for bivariate Archimedean copulas. (English) Zbl 0785.62032

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Summary: A bivariate distribution function $H(x, y)$ with marginals $F(x)$ and $G(y)$ is said to be generated by an Archimedean copula if it can be expressed in the form

$$H(x, y) = \varphi^{-1}[\varphi\{F(x)\} + \varphi\{G(y)\}]$$

for some convex, decreasing function φ defined on $(0, 1]$ in such a way that $\varphi(1) = 0$. Many well-known systems of bivariate distributions belong to this class, including those of Gumbel, Ali-Mikhail-Haq-Thélot, Clayton, Frank, and Hougaard. Frailty models also fall under that general prescription.

This article examines the problem of selecting an Archimedean copula providing a suitable representation of the dependence structure between two variates X and Y in the light of a random sample $(X_1, Y_1), \ldots, (X_n, Y_n)$. The key to the estimation procedure is a one- dimensional empirical distribution function that can be constructed whether the uniform representation of X and Y is Archimedean or not, and independently of their marginals. This semiparametric estimator, based on a decomposition of Kendall's tau statistic, is seen to be $\sqrt{n}$-consistent, and an explicit formula for its asymptotic variance is provided. This leads to a strategy for selecting the parametric family of Archimedean copulas that provides the best possible fit to a given set of data. To illustrate these procedures, a uranium exploration data set is reanalyzed. Although the presentation is restricted to problems involving a random sample from a bivariate distribution, extensions to situations involving multivariate or censored data could be envisaged.

MSC:

62G05 Nonparametric estimation
62G30 Order statistics; empirical distribution functions
62G07 Density estimation

Keywords:

empirical process; Frailty model; U statistic; Archimedean copula; bivariate distributions; dependence structure; empirical distribution; semiparametric estimator; decomposition of Kendall's tau statistic; asymptotic variance; uranium exploration data set

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