Takeuti, Gaisi

RSUV **isomorphisms.** (English) Zbl 0792.03041

Clote, Peter (ed.) et al., Arithmetic, proof theory, and computational complexity. Oxford: Clarendon Press. Oxf. Logic Guides. 23, 364-386 (1993).

In his book: Bounded arithmetic (1986; Zbl 0649.03042), *S. Buss* introduced the systems S_3^i, S_2^i, $V_2^i(BD)$, and $U_2^i(BD)$, where S_2^1, $V_2^1(BD)$ and $U_2^1(BD)$ correspond to polynomial time computable functions, exponential time computable functions, and polynomial space computable functions, respectively, and S_3^i is a generalization of S_2^i by introducing $\#_3$ with the intended property $a\#_3 b = 2^{|a|\#|b|}$. In a previous paper [Arch. Math. Logic 29, No. 3, 149-169 (1990; Zbl 0681.03040)], the author established an interpretation of $V_2^i(BD)$ in S_3^i and an inverse interpretation of S_3^i in $V_2^i(BD)$. In the present paper the isomorphisms between $V_2^i(BD)$ and S_3^i and between R_3^i and $U_2^i(BD)$ are proved, where R_3^i is obtained from R_2^i by introducing $\#_3$ and R_2^1 is equivalent to the system of *B. Allen* [Ann. Pure Appl. Logic 53, No. 1, 1-50 (1991; Zbl 0741.03019)] and also to P. Clote's system TNC [*P. Clote* and the author, Ann. Pure Appl. Logic 56, No. 1-3, 73-117 (1992; Zbl 0772.03028)], both of which correspond to NC.

These isomorphisms are also extended in the paper to the isomorphisms between R_{k+1}^i and $U_k^i(BD)$ and between S_{k+1}^i and $V_k^i(BD)$, where R_{k+1}^i, S_{k+1}^i, $U_{k+1}^i(BD)$, and $V_{k+1}^i(BD)$, respectively, by introducing $\#_{k+1}$ with the intended property $a\#_{k+1}b = 2^{a\#_k b}$. The isomorphism between S_{k+1}^i and $V_k^i(BD)$ is also independently proved by *A. A. Razborov* ["An equivalence between second order bounded domain bounded arithmetic and first order bounded a arithmetic", P. Clote (ed.) et. al.: Arithmetic, proof theory, and computational complexity, Oxford Logic Guides 23, 247- 277 (1993)]. In the paper the author also proved conservative rsults between $U_2^i(BD)$ and $\overline{U}_2^i(BD)$, where $\overline{U}_2^i(BD)$ is obtained from $U_2^i(BD)$ by introducing $\Delta_i^{1,b}$-comprehension schema.

Finally it is shown that V_2^{i+1} proves $\forall x \le a \exists y (A(x,y) \wedge |y| \le b) \to \exists t \forall x \le a \exists y \le t A(x,y)$, where $A(x,y)$ is a Σ_i^b-formula and V_2^{i+1} is also Buss' system closely related to $V_2^{i+1}(BD)$.

For the entire collection see [Zbl 0777.00008].

Reviewer: G.Takeuti (Urbana, IL)

MSC:

03F30 First-order arithmetic and fragments

Keywords:

computable functions; bounded arithmetic; isomorphism of theories