Horadam, K. J.; **de Launey, W.**
Cocyclic development of designs. (English) Zbl 0785.05019
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Summary: We present the basic theory of cocyclic development of designs, in which group development over a finite group G is modified by the action of a cocycle defined on $G \times G$. Negacyclic and ω-cyclic development are both special cases of cocyclic development.

Techniques of design construction using the group ring, arising from difference set methods, also apply to cocyclic designs. Important classes of Hadamard matrices and generalized weighing matrices are cocylic.

We derive a characterization of cocyclic development which allows us to generate all matrices which are cocyclic over G. Any cocyclic matrix is equivalent to one obtained by entrywise action of an asymmetric matrix and a symmetric matrix on a G-developed matrix. The symmetric matrix is a Kronecker product of back ω-cyclic matrices, and the asymmetric matrix is determined by the second integral homology group of G. We believe this link between combinatorial design theory and low-dimensional group cohomology leads to (i) a new way to generate combinatorial designs; (ii) a better understanding of the structure of some known designs; and (iii) a better understanding of known construction techniques.

MSC:

05B20 Combinatorial aspects of matrices (incidence, Hadamard, etc.)
05B10 Combinatorial aspects of difference sets (number-theoretic, group-theoretic, etc.)
05B15 Orthogonal arrays, Latin squares, Room squares
20J05 Homological methods in group theory
20J06 Cohomology of groups

Keywords:

orthogonal design; negacyclic development; extension group; cocyclic development of designs; ω-cyclic development; difference set; Hadamard matrices; weighing matrices; cocyclic matrix; homology group

Full Text: DOI

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