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A unified approach to the tau method and Chebyshev series expansion techniques. (English)

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The authors consider problems with differential equations of the following kind: (1) $Dy(x) := \sum_{r=0}^{\nu} p_r(x)y^{(r)}(x) = f(x)$, $x \in [-1,1]$ and (2) $B_i(y) = \gamma_i$, $\gamma_i =$ constant, $i = 1,\dots,\nu$, where f is a polynomial in x of some given degree, the p_r are polynomial coefficients and B_i, $i = 1,\dots,\nu$, are linear functionals defined on $C^{\nu}[-1,1]$: $B_i(y) = \sum_{k=0}^{\nu-1}\sum_{j=1}^{\mu} \beta_{ij}^{(k)} y^{(k)}(a_j)$, $i = 1,\dots,\nu$, where $a_1,\dots,a_\mu$ are fixed real numbers in $[-1,1]$ and $\beta_{ij}^{(k)}$ are some given constants.

The essential idea of the Lanczos tau method is to perturb problem (1), (2) in such a way that its exact solution becomes a polynomial, i.e. solving exactly the perturbed problem (3) $Dy_n(x) = f(x) + H_n(x)$, $x \in [-1,1]$, $n \in \mathbb{N}$, $B_i(y_n) = \gamma_i$, $i = 1,\dots,\nu$, where H_n is a polynomial perturbation term which reduces the exact solution of (3) to a polynomial y_n which will be called the nth tau approximation of y.

Two main choices of $H_n(x)$ are considered in this paper: $H_n(V) := \sum_{i=1}^{r-1} \tau_i V_{n+1}(x)$ and $H_n(x,V) := \left(\sum_{i=0}^{r-1} \tau_i x^i\right) V_n(x)$, where V_k are polynomials of degree k in x.

The authors show the equivalence between the above recursive formulation of the tau method and an alternative technique called the operational approach because it reduces differential problems to linear algebraic problems. The authors then use such methods as analytic tools in the simulation of two classical numerical techniques based on Chebyshev series expansions. The results make possible the recursive formulation of a variety of series expansion methods.

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MSC:

65L10	Numerical solution of boundary value problems involving ordinary differential equations
34B10	Nonlocal and multipoint boundary value problems for ordinary differential equations

Keywords:

Lanczos tau method; polynomial perturbation; Chebyshev series expansions; series expansion methods

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paper as accurately as possible without claiming the completeness or perfect precision of the matching.