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On graphs having σ-polynomials of the same degree. (English) Zbl 0771.05038

Discrete Math. 110, No. 1-3, 185-196 (1992).

The graphs considered in this paper are finite and undirected, with no loops or parallel edges. The σ-polynomial $\sigma(G,t)$ of a graph G with p vertices is defined in *R. R. Korfhage* [σ- polynomials and graph coloring, J. Comb. Theory, Ser. B 24, No. 2, 137- 153 (1978)] as follows: if the chromatic polynomial $p(G,\lambda)$ of G is $\sum_{i=0}^{p-\chi(G)} a_i\lambda(\lambda-1)\cdots(\lambda-(p-i)+1)$, where $\chi(G)$ is the chromatic number of G, then $\sigma(G,t)=\sum_{i=0}^{p-\chi(G)} a_i t^{p-\chi(G)-i}$. A theorem in *R. C. Read* [An introduction to chromatic polynomials, J. Comb. Theory 4, 52-71 (1967; Zbl 0173.262)] which identifies a_i as the number of subgraphs of the complement of G which are isomorphic to the union of complete graphs with a total of i vertices, is used to obtain a necessary and sufficient condition for the degree $p-\chi(G)$ of $\sigma(G,t)$ to be k for any positive integer k. This generalizes the condition found in the above-mentioned paper of R. R. Korfhage for $k=0$ and 1 and in *M. Dhurandhar* [J. Comb. Theory, Ser. B 37, 210- 220 (1984; Zbl 0554.05030)]. This condition is then used to construct all the graphs whose σ-polynomials are of degree 2, 3 and 4; the results for degree 2 agree with those in *R. W. Frucht* and *R. E. Giudici* [Ars. Comb. 16-A, 161-172 (1983; Zbl 0536.05026)].

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