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On orthogonal polynomials of Sobolev type: Algebraic properties and zeros. (English)

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The authors study orthogonal polynomials with respect to a special Sobolev inner product given by

$$\langle f, g \rangle = \int_I f(x)g(x)d\mu(x) + Mf(c)g(c) + Nf'(c)g'(c),$$

with $c \in \mathbb{R}$ and $M, N \geq 0$. The emphasis is on algebraic properties, such as a representation of the Sobolev orthogonal polynomials $Q_n(x)$ in terms of orthogonal polynomials related to the measure μ and polynomial modifications of it. A five-term recurrence relation for $Q_n(x)$ is given and formulas for the reproducing kernels are derived. A large section deals with various properties of the zeros of the Sobolev orthogonal polynomials. Particular interest is given to the symmetric case, i.e., the case when both the interval I and the measure μ are symmetric. The authors also examine differential properties of the polynomials $Q_n(x)$ for the Sobolev modification. Some examples are worked out, e.g., the cases when the polynomials $P_n(x)$ are Gegenbauer polynomials and Poisson-Charlier polynomials.

Reviewer: [W. Van Assche \(Heverlee\)](#)

MSC:

33C45 Orthogonal polynomials and functions of hypergeometric type (Jacobi, Laguerre, Hermite, Askey scheme, etc.)

Cited in **2** Reviews
Cited in **44** Documents

Keywords:

Sobolev inner product; reproducing kernels; Gegenbauer polynomials; Poisson-Charlier polynomials

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