

**Franks, John**

**Geodesics on  $S^2$  and periodic points of annulus homeomorphisms.** (English) Zbl 0766.53037  
*Invent. Math.* 108, No. 2, 403–418 (1992).

It is the main result of this paper that an area preserving homeomorphism of the open or closed annulus which has at least one periodic point has infinitely many interior periodic points. It was shown by Birkhoff that closed geodesics on the 2-sphere with a metric of positive Gaussian curvature can be described as periodic points of an area preserving annulus map. Together with recent work by Victor Bangert it follows from the main result of this paper that for every Riemannian metric on the 2- sphere there are infinitely many closed geodesics.

Reviewer: [H.-B.Rademacher \(Bonn\)](#)

**MSC:**

[53C22](#) Geodesics in global differential geometry  
[37G99](#) Local and nonlocal bifurcation theory for dynamical systems

Cited in **7** Reviews  
Cited in **82** Documents

**Keywords:**

area preserving homeomorphism; annulus; periodic point; closed geodesics

**Full Text:** [DOI](#) [EuDML](#)

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