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**Galois group over  $\mathbb{Q}$  of some iterated polynomials.** (English) [Zbl 0758.11045](#)  
*Arch. Math.* 59, No. 3, 239-244 (1992).

Let  $a$  be an integer such that  $-a$  is not a square in  $\mathbb{Q}$ ,  $f := X^2 + a \in \mathbb{Z}[X]$ , and denote the iterates of  $f$  by  $f_0 := X$  and  $f_{n+1} := f(f_n) = f_n^2 + a$  for all  $n \geq 0$ . Let  $c_1 := -a$  and  $c_{n+1} := f(c_n) = c_n^2 + a$  for  $n \geq 1$ . There is an integer sequence  $(b_n)_{n \geq 1}$  with the  $b_n$  coprime in pairs such that for all  $n \geq 1$ ,  $c_n = \prod_{d|n} b_d$ . Let  $K_n$  be the splitting field of  $f_n$  over  $\mathbb{Q}$  and denote by  $\Omega_n := \text{Gal}(K_n/\mathbb{Q}) = \text{Gal}(f_n/\mathbb{Q})$  its Galois group over the rational numbers. Let  $[C_2]^n$  denote the  $n$ -fold wreath product of the 2-element group. Then it is known that  $\Omega_n$  always injects into  $[C_2]^n$ . The following equivalence is shown in the paper:  $\Omega_n \cong [C_2]^n$  if and only if  $c_1, c_2, \dots, c_n$  are 2-independent in  $\mathbb{Q}$ .

Here, "2-independent" means that no nonempty product of some of the  $c_n$  is a square. This gives the following sufficient condition for  $\Omega_n \cong [C_2]^n$  to hold:  $|b_m|$  is not a square for  $2 \leq m \leq n$ .

This condition is then verified for all  $n$  in the following cases: ( $a > 0$  and  $a \equiv 1$  or  $2 \pmod{4}$ ) or ( $a < 0$  and  $a \equiv 0 \pmod{4}$  and  $-a$  not a square). So, for these  $a$ , one always has  $\text{Gal}(f_n/\mathbb{Q}) \cong [C_2]^n$  for all  $n$ .

Reviewer: [M.Stoll \(München\)](#)

**MSC:**

**11R32** Galois theory  
**11R09** Polynomials (irreducibility, etc.)  
**11B37** Recurrences

Cited in **3** Reviews  
Cited in **23** Documents

**Keywords:**

iterated polynomials; recursion; Galois group; wreath product of the 2- element group

**Full Text:** [DOI](#)

**References:**

- [1] J. E. Cremona, On the Galois groups of the iterates of  $x^2+1$ . *Mathematika*36, 259-261 (1989). · [Zbl 0699.12018](#) · [doi:10.1112/S0025579300013127](#)
- [2] R. W. K. Odoni, Realising wreath products of cyclic groups as Galois groups. *Mathematika*35, 101-113 (1988). · [Zbl 0662.12010](#) · [doi:10.1112/S002557930000632X](#)

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