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Two parallel processors with coupled inputs (with an afterword by Alan Weiss). (English)

Zbl 0760.60093
Adv. Appl. Probab. 24, No. 4, 986-1007 (1992).

Summary: We consider the double queue arising from a system consisting of two processors serving three job streams generated by independent Poisson sources. The central job stream of rate ν consists of jobs which place resource demands on both processors, which are handled separately by each processor once the request is made. In addition, the first processor receives background work at a rate of λ while the second receives similar tasks at a rate η. Each processor has exponentially distributed service times with rates α and β, respectively. A functional equation is found for $P(z,w)$, the generating function of the joint queue-length distribution, which leads to a relation between $P(z,0)$ and $P(0,w)$ in the region $|z|, |w| < 1$ of a complex algebraic curve associated with the problem. The curve is parametrized by elliptic functions $z(\xi)$ and $w(\xi)$ and the relation between $P(z(\xi),0)$ and $P(0,w(\xi))$ persists on their analytic continuation as elliptic functions in the ξ-plane. This leads to their eventual determination by an appeal to the theory of elliptic functions. From this determination we obtain asymptotic limit laws for the expectations of the mean number of jobs in each queue conditioned on the other, as the number of jobs in both processors tends to ∞. Transitions are observed in the asymptotic behavior of these quantities as one crosses various boundaries in the parameter space. An interpretation of these results via the theory of large deviations is presented.

MSC:

60K25 Queueing theory (aspects of probability theory)
90B22 Queues and service in operations research

Keywords:

double queue; two processors serving three job streams; functional equation; analytic continuation; asymptotic limit laws for the expectations of the mean number of jobs in each queue; large deviations

Full Text: DOI