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Train tracks and automorphisms of free groups. (English) Zbl 0757.57004

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Let G be a graph whose fundamental group has been identified with the free group F_n and let $f : G \to G$ be a homotopy equivalence. Then f is said to be a train track map if f maps vertices to vertices and the restriction of f^k to the interior of every edge of G is locally injective for all $k > 0$. An outer automorphism A of F_n is reducible if there are proper free factors $F_1, \ldots, F_k$ of F_n with $F_1 * \cdots * F_k$ a free factor of F_n and such that A transitively permutes the conjugacy classes of the F_i's. Otherwise A is irreducible.

The authors give a constructive proof of the following conjecture of Thurston: Every irreducible outer automorphism of F_n is topologically represented by a train track map.

The idea of the proof is to change a topological representative of A by Stallings-folds, tightenings, subdivisions, and collapsing of forests. As an application it is shown that if $\varphi : F_n \to F_n$ is an automorphism in an irreducible automorphism class, then Rank (Fixφ) ≤ 1.

The authors then define stable relative train track maps and obtain the main result of the paper: Every outer automorphism A of F_n can be represented by a stable relative train track map $f : G \to G$. As an application the following conjecture of Scott is proved: For any automorphism $\varphi : F_n \to F_n$, Rank(Fix(φ)) $\leq n$.

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MSC:

57M07	Topological methods in group theory
20E05	Free nonabelian groups
20F34	Fundamental groups and their automorphisms (group-theoretic aspects)
20E08	Groups acting on trees
20F65	Geometric group theory

Keywords:

ranks of fixed point subgroups; graphs; fundamental groups; free groups; train track maps; outer automorphisms; reducible automorphisms; irreducible outer automorphisms; stable relative train track maps

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