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On the residuality of the set of norms having Mazur's intersection property. (English)

Zbl 0760.46007

Math. Balk., New Ser. 5, No. 1, 20-26 (1991).

A Banach space (E, p) has Mazur's intersection property if every closed bounded convex subset of E is the intersection of the closed balls that contain it. Similarly, if p^* is the norm on E^* dual to p , (E^*, p^*) has w^* -Mazur's intersection property if every w^* -compact convex subset of E^* is the intersection of all the closed balls that contain it.

The author continues his own work on the subject [Proc. Am. Math. Soc. 104, No. 1, 157-164 (1988; Zbl 0673.46005)] by using a well-constructed Baire category argument to show that if $(E, \|\cdot\|)$ has Mazur's intersection property and P is the set of all norms on E equivalent to $\|\cdot\|$ (in the usual uniform topology of real-valued functions on the unit ball of E), then P has a dense G_δ -subset P_0 such that (E, p) has Mazur's intersection property for each p in P_0 . The analogous theorem for w^* -Mazur's intersection property for dual norms on E^* is also true, by (essentially) the same proof. Finally, when (E, p) has Mazur's intersection property and (E^*, p^*) has w^* -Mazur's intersection property, where p^* is the dual norm of p , then p and p^* are Fréchet differentiable on dense G_δ -subsets of E and E^* respectively; this involves deep results from convex analysis. The author concludes by discussing various known results from the viewpoint of the proofs in this paper.

Reviewer: [S.J.Sidney \(Storrs\)](#)

MSC:

[46B03](#) Isomorphic theory (including renorming) of Banach spaces

[46B20](#) Geometry and structure of normed linear spaces

Cited in **3** Documents

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Mazur's intersection property; Baire category argument; w^* -Mazur's intersection property for dual norms; Fréchet differentiable