Freedman, M. A.

Further investigation of the relation of the operator $\partial/\partial\sigma + \partial/\partial\tau$ to evolution governed by accretive operators. (English) Zbl 0811.35172

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From the introduction: Crandall and Evans proved existence of mild solution $u(t)$ to the nonlinear Cauchy problem

$$\frac{du(t)}{dt} + Au(t) \ni f(t) \quad (0 \le t \le T), \quad u(0) = x_0 \in \overline{\text{Dom}(A)}. \tag{1}$$

Here A is a multivalued accretive operator on a Banach space X and $f \in L^1(0,T;X)$. By a mild solution, we mean u is a limit of difference approximations to (1). We obtain a mild solution to the Cauchy problem

$$\frac{du(t)}{dt} + Au(t) \ni F\big(t, u(t)\big) \quad (0 \le t \le T), \quad u(0) = x_0 \in \overline{\text{Dom}(A)} \tag{2}$$

with A, as in (1), accretive on X. (2) includes the case $F = f$ of (1) and other cases that are of greater complexity. Informally, given functions ω and h, consider the partial differential equation on $(0,S] \times (0,T]$

$$\frac{\partial v}{\partial \sigma}(\sigma,\tau) + \frac{\partial v}{\partial \tau}(\sigma,\tau) = h(\sigma,\tau) \tag{3}$$

with boundary condition $v(\sigma,\tau) = \omega(\tau - \sigma)$, when $\sigma\tau = 0$. For each positive integer m and n let $v_{m,n}$ be the difference approximation to v obtained when $\partial\sigma = 1/m$ and $\partial\tau = 1/n$ in (3). Likewise, in either (1) or (2), let u_n denote the difference approximation to u when dt is replaced by $1/n$. Then for the right choice of ω and h – which will depend on f in (1) and on F in (2), an estimate of $u_m - u_n$ in terms of $v_{m,n}$ can be obtained which forces $\|u_m - u_n\|$ to zero as $v_{m,n}$ converges to v. In this way $\{u_n\}_n$ is shown to be Cauchy and the mild solution $u = \lim_n u_n$ is shown to exist. The emphasis in this work is directed towards methods and estimates tied to convergence of $v_{m,n}$ to v. Another paper based on the theory presented here will be devoted to the study of the nonautonomous Cauchy equation $du/dt + A(t) \ni 0$.

MSC:

35R70 PDEs with multivalued right-hand sides
35G10 Initial value problems for linear higher-order PDEs
35K25 Higher-order parabolic equations
47J05 Equations involving nonlinear operators (general)
47H06 Nonlinear accretive operators, dissipative operators, etc.

Keywords:

nonlinear Cauchy problem; multivalued accretive operator; mild solution; difference approximations