Dang, T.; **Friedman, Y.**; **Russo, B.**

Affine geometric proofs of the Banach Stone theorems of Kadison and Kaup. (English)

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Rocky Mt. J. Math. 20, No. 2, 409-428 (1990).

R. V. Kadison, Ann. Math. (2) 54, 325–338 (1951; Zbl 0045.06201), proved that a surjective linear isometry T between two unital C^*-algebras A, B is of the form $Tx = u \cdot \rho(x)$, $x \in A$, where u is a unitary element of B and $\rho : A \to B$ is a Jordan isomorphism. Using the complicated machinery of infinite dimensional holomorphy, *W. Kaup*, Math. Ann. 228, 39–64 (1977; Zbl 0335.58005), extended this result proving that every surjective linear isometry T between two JB^*-triples is a JB^*-triple isomorphism.

The aim of this paper is to give an elementary proof of Kaup's result based on the affine geometric properties of faces (= extremal convex subsets) in the state space together with analogs of standard operator tools as spectral, polar and Jordan decompositions, biduals and a theorem of Effros.

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MSC:

47C15	Linear operators in C^*- or von Neumann algebras
46E40	Spaces of vector- and operator-valued functions
58B20	Riemannian, Finsler and other geometric structures on infinite-dimensional manifolds

Keywords:

Banach Stone theorem; Jordan isomorphism; infinite dimensional holomorphy; surjective linear isometry; JB^*-triples; JB^*-triple isomorphism; affine geometric properties of faces; extremal convex subsets; state space; spectral, polar and Jordan decompositions; biduals; theorem of Effros

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