Jaeger, François; Vertigan, D. L.; Welsh, D. J. A.
On the computational complexity of the Jones and Tutte polynomials. (English)
Zbl 0747.57006
Math. Proc. Camb. Philos. Soc. 108, No. 1, 35-53 (1990).

From the introduction: "The original problem motivating this paper is to decide whether or not computing the Jones polynomial of a link is, in general, a feasible computation. To put this question into perspective, it is well known that computation of the Alexander-Conway polynomial of a link is 'easy' (that is, can be done in time polynomial in the size of the input) being just the expansion of a one-variable determinant, but that the computations of the Homfly and Kauffman polynomials of a link are NP-hard. The Jones polynomial could be regarded as lying somewhere between the Alexander-Conway polynomial and these two other link polynomials in terms of computational difficulty. However, as we shall see, it turns out to be computationally intractable in a very strong sense, even for the special case of alternating links." "We show that determining the Jones polynomial of an alternating link is $\#P$-hard. This is a special case of a wide range of results on the general intractability of the evaluation of the Tutte polynomial of a matroid".

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MSC:

57M25 Knots and links in the 3-sphere (MSC2010)
57M15 Relations of low-dimensional topology with graph theory
68Q25 Analysis of algorithms and problem complexity
68R10 Graph theory (including graph drawing) in computer science

Keywords:

computing the Jones polynomial of a link; Alexander-Conway polynomial; $\#P$-hard; Tutte polynomial of a matroid

Full Text: DOI

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