Scholl, A. J.

Motives for modular forms. (English) Zbl 0760.14002

Invent. Math. 100, No. 2, 419-430 (1990).

P. Deligne defined in Sémin. Bourbaki 1968/69, Exp. No. 355, Lect. Notes Math. 179, 139–172 (1971; Zbl 0206.49901) the ℓ-adic parabolic cohomology groups attached to holomorphic cusp forms of weight ≥ 2 on congruence subgroups of $\mathrm{SL}_2(\mathbb{Z})$ as certain subgroups in the ℓ-adic cohomology of Kuga-Sato varieties. In the paper under review, the author constructs these groups as the kernel of certain projectors in the Chow ring of algebraic correspondences modulo rational equivalence. Then the author calls a Grothendieck motive (with coefficients in a number field L) an object of the category of motives over $\mathbb{Q}$ in which $\mathrm{Hom}(h(Y), h(X))$ is the group of algebraic cycles on $X \times Y$ of codimension $\dim Y$, tensored with L, modulo homological equivalence. The following theorem is proved: If $f = \sum_{m=1}^{\infty} a_m q^m$ is a normalized newform of weight w, level n and character χ, and L is the field generated by the coefficients a, then there is a Grothendieck motive $M(f)$ over $\mathbb{Q}$ with coefficients in L such that if $p \not| nl$, and λ is a prime of L dividing ℓ, then the λ-adic realization $H_\lambda(M(f))$ of $M(f)$ uniramified at p, and the characteristic polynomial of a geometric Frobenius at p is the Hecke polynomial $T_p(X) = X^2 + a_p X + chi(p)p^{w-1}$.

A relation between the p-adic representation of $\mathrm{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$ attached to a modular form, and the action of the Hecke operator T_p is derived as application.

Reviewer: A.M.Shermenev (Moskva)

MSC:

14A20	Generalizations (algebraic spaces, stacks)
14F30	p-adic cohomology, crystalline cohomology
14G35	Modular and Shimura varieties
11G18	Arithmetic aspects of modular and Shimura varieties

Keywords:

ℓ-adic parabolic cohomology groups; Kuga-Sato varieties; Grothendieck motive; Hecke polynomial

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