

Reshetikhin, N. Yu.; Turaev, V. G.

Ribbon graphs and their invariants derived from quantum groups. (English) Zbl 0768.57003
Commun. Math. Phys. 127, No. 1, 1-26 (1990).

The authors construct the generalization of the Jones polynomial of links to the case of graphs in R^3 . They introduce the so-called coloured ribbon graphs in $R^2 \times [0, 1]$ and define for them Jones-type isotopy invariants. The approach to colouring is based on Drinfeld's notion of a quasitriangular Hopf algebra. For each quasitriangular Hopf algebra A the authors define A -coloured (ribbon) graphs. The colour of an edge is an A -module. The colour of a vertex is an A -linear homomorphism intertwining the modules which correspond to edges incident to this vertex. The category of an A -coloured ribbon graph is a compact braided strict monoidal category introduced by *A. Joyal* and *R. Street* [Braided monoidal categories, Macquarie Math. Reports, Report No. 860081 (1986)]. If A satisfies a minor additional condition, then the authors construct a canonical covariant functor from the category of A -coloured ribbon graphs into the category of A -modules. In the case of A being the quantized universal enveloping algebra of sl_2 this functor generalizes the Jones polynomial of links. If $A = U_h(sl_n(C))$ this generalizes the Jones-Conway (Thomflyp) polynomial and for $A = U_h G$, $G = so(n)$, $sp(2k)$ the Kauffman polynomial. The paper under review was followed by the authors' paper "Invariants of 3-manifolds via link polynomials and quantum groups" [Invent. Math. 103, 547-597 (1991; Zbl 0725.57007)] in which the authors construct new topological invariants of compact oriented manifolds (employing the methods of the paper under review). The construction was partially inspired by ideas of *E. Witten* [Quantum field theory and the Jones polynomial, *Commun. Math. Phys.* 121, 351-399 (1989; Zbl 0667.57005)] who considered quantum field theory defined by the nonabelian Chern-Simon action and applied it to the study of 3-manifolds (on physical level of rigor).

Reviewer: [J.H.Przytycki \(Riverside\)](#)

MSC:

- 57M25 Knots and links in the 3-sphere (MSC2010)
- 81R05 Finite-dimensional groups and algebras motivated by physics and their representations
- 17B37 Quantum groups (quantized enveloping algebras) and related deformations
- 81T40 Two-dimensional field theories, conformal field theories, etc. in quantum mechanics
- 16W30 Hopf algebras (associative rings and algebras) (MSC2000)

Cited in **24** Reviews
Cited in **340** Documents

Keywords:

generalization of the Jones polynomial; graphs in R^3 ; coloured ribbon graphs; quasitriangular Hopf algebra; quantized universal enveloping algebra of sl_2 ; Kauffman polynomial; quantum field theory

Full Text: [DOI](#)

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